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A weighted measure for the similarity analysis of DNA sequences

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Abstract Here we propose a weighted measure for the similarity analysis of DNA sequences. It is based on LZ complexity and (0,1) characteristic sequences of DNA sequences. This weighted measure enables biologists to extract similarity information from biological sequences according to their requirements. For example, by this weighted measure, one can obtain either the full similarity information or a similarity analysis from a given biological aspect. Moreover, the length of DNA sequence is not problematic. The application of the weighted measure to the similarity analysis of β -globin genes from nine species shows its flexibility.

Keywords DNA sequence · (0,1) characteristic sequence · LZ complexity · Weighted measure

Introduction

With the completion of more genome projects, the list of genome sequences is expanding rapidly, which urges scientists to analyze them as far as possible so as to obtain more information about them. The most common method used to analyze DNA sequences is sequence alignment. This procedure generates a matrix of numbers that represents all possible alignments between the sequences. The highest set of sequential scores in the matrix defines an optimal alignment. For DNA sequences, a nucleotide-

substitution matrix is used to score matches and mismatches. The advantage of using these matrices is that they are based on a defined evolutionary model and that the statistical significance of alignment scores obtained by local-alignment programs may be evaluated. However, their function is only realized with the aid of dynamic programming, which will be slow due to the large number of computational steps, and hence is not easy to use for long sequences [1]. Another method that has become popular recently relies on the quantitative characterization of DNA sequences, where many indices/invariants/descriptors have been calculated for the similarity analysis. This method is based on the fact that these indices/invariants/descriptors are a kind of characterization indices of DNA sequences. For instance, Raychaudhury and Nandy stated that graph radius g_R could effectively differentiate sequences of different size [2]. Randić et al. proposed that the leading eigenvalues of M/M and L/L matrices are invariants/descriptors that relate to DNA sequences, where the entries of the M/M matrix are given as a quotient of the Euclidean distance between two vertices of the zigzag curve of DNA sequences and the number of edges between two vertices. The entries of the L/L matrix are given as a quotient of the Euclidean distance between two vertices of the zigzag curve of DNA sequences and the sum of geometrical lengths of edges between the two vertices [3]. Work associated with invariants/descriptors can also be found in [4–9]. However, as pointed out in [4], this approach involves a series of as yet unresolved questions, which includes the questions as follows: (1) how to obtain and select suitable invariants/descriptors to characterize DNA sequences so as to compare sequences effectively; (2) the calculation of some effective invariants/descriptors become more and more difficult with the length of the sequences getting more large; (3) there is some loss of information.

In this paper, we propose a weighted measure for the similarity analysis of DNA sequences based on the (0,1) characteristic sequences and LZ complexity. The famous LZ complexity is related to the number of steps in a self-delimiting production process by which a given sequence is

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presumed to be generated [10]. The virtue of this measure is its flexibility. Three weights make it convenient to emphasize a given aspect of DNA characteristics, which is useful to biologists to further analyze DNA sequences from different aspects. At the end of this paper, we illustrate the application of this weighted measure.

Special similarity measure

Let S, Q and R be sequences over a finite alphabet Δ , $L(S)$ be the length of S , $S(i)$ be the i th element of S and $S(i,j)$ be the subsequence of S that starts at position i and ends at position j . Note that $S(i,j)=\Phi$, for $i>j$. The concatenation of Q and R forms a new sequence $S=QR$, where Q is called a prefix of S , and S is called an extension of Q if there exists an integer i such that $Q=S(1,i)$.

An extension $S=QR$ of Q is reproducible from Q denoted by $Q \rightarrow S$, if there exists an integer $p \leq L(Q)$ such that $R(k) = S(p+k-1)$, for $k=1, 2, \dots, L(R)$. For example: AA-GA \rightarrow AAGAAGA, with $p=2$.

A non-null sequence S is producible from its prefix $S(1,j)$, denoted by $S(1,j) \rightarrow S$, if $S(1,j) \rightarrow S(1, L(S)-1)$. For example: AGGA \rightarrow AGGAGGG with $p=2$.

Any non-null sequence S can be built from a production process by an iterative self-deleting-building process where at the i th step $S(1, h_{i-1}) \rightarrow S(1, h_i)$, $\Phi = S(1, 0) \rightarrow S(1, 1)$. An m -step production process of S leads to a parsing of S into $H(S) = S(1, h_1) \cdot S(h_1 + 1, h_2) \cdot \dots \cdot S(h_{m-1} + 1, h_m)$, which is called the history of S , and $H_i(S) = S(h_{i-1} + 1, h_i)$ is called the i th component of $H(S)$.

A component $H_i(S)$ is called exhaustive if $S(1, h_{i-1}) \rightarrow S(1, h_i)$ is not true. A history is called exhaustive if each of its components (with a possible exception of the last one) is exhaustive. What is more important, the exhaustive history of any non-null sequence is unique. For example, for the sequence $S = \text{TGGGTTGGTTTG}$, its exhaustive history is $\text{EH}(S) = \text{T} \cdot \text{G} \cdot \text{GGT} \cdot \text{TGGT} \cdot \text{TTG}$.

Let $c(S)$ be the number of components in the exhaustive history of S , then it is an important complexity indicator because the production process here is an example of a class of parsing rules. According to [10], for any given sequences S and Q , the following property always remains valid: $c(QS) \leq c(Q) + c(S)$. Furthermore, Otu and Sayood [11] have proposed that the more similar S is to Q , the smaller $c(QS) - c(Q)$ is. That is $c(QS) - c(Q)$ depends on how much S is similar to Q .

For example, let S, Q, R represent three short DNA sequences defined over the set $\{A, C, G, T\}$. $S = \text{ATTCTGAGG TACGTAAAG}$, $Q = \text{GGTCTGATCTAGAACGTA}$, $R = \text{TAG CCACGATGCAGAC}$. By the above mentioned rule, the corresponding exhaustive histories of $S, Q, R, SQ, SR, QR, QS, RS$ are: $\text{A} \cdot \text{T} \cdot \text{TC} \cdot \text{TG} \cdot \text{AG} \cdot \text{GT} \cdot \text{AC} \cdot \text{GTAA} \cdot \text{AG}$, $\text{G} \cdot \text{GT} \cdot \text{C} \cdot \text{TG} \cdot \text{A} \cdot \text{TCTA} \cdot \text{GAA} \cdot \text{CG} \cdot \text{TA}$, $\text{T} \cdot \text{A} \cdot \text{G} \cdot \text{C} \cdot \text{CA} \cdot \text{CG} \cdot \text{AT} \cdot \text{GC} \cdot \text{A} \cdot \text{GAC}$, $\text{A} \cdot \text{T} \cdot \text{TC} \cdot \text{TG} \cdot \text{AG} \cdot \text{GT} \cdot \text{AC} \cdot \text{GTAA} \cdot \text{AGGG} \cdot \text{TCTG}$, $\text{AT} \cdot \text{CTA} \cdot \text{GAA} \cdot \text{CGTA}$, $\text{A} \cdot \text{T} \cdot \text{TC} \cdot \text{TG} \cdot \text{AG} \cdot \text{GT} \cdot \text{AC} \cdot \text{GTAA} \cdot \text{AGT} \cdot \text{AGC} \cdot \text{CA} \cdot \text{CGA} \cdot \text{TGC} \cdot \text{AGA} \cdot \text{C}$, $\text{G} \cdot \text{GT} \cdot \text{C} \cdot \text{TG} \cdot \text{A} \cdot \text{TC}$

$\text{TA} \cdot \text{GAA} \cdot \text{CG} \cdot \text{TAT} \cdot \text{AG} \cdot \text{CC} \cdot \text{ACGA} \cdot \text{TGC} \cdot \text{AGA} \cdot \text{C}$, $\text{G} \cdot \text{GT} \cdot \text{C} \cdot \text{TG} \cdot \text{A} \cdot \text{TCTA} \cdot \text{GAA} \cdot \text{CG} \cdot \text{TAA} \cdot \text{TT}$, $\text{CTGAG} \cdot \text{GTAC} \cdot \text{GTAAA} \cdot \text{G}$, $\text{T} \cdot \text{A} \cdot \text{G} \cdot \text{C} \cdot \text{CA} \cdot \text{CG} \cdot \text{AT} \cdot \text{GCA} \cdot \text{GAC} \cdot \text{ATT} \cdot \text{CT} \cdot \text{GA} \cdot \text{G} \cdot \text{GT} \cdot \text{ACGT} \cdot \text{AA} \cdot \text{AG}$.

So $C(S)=9, C(Q)=9, C(R)=9, C(SQ)=13, C(SR)=15, C(QS)=14, C(RS)=16$. We find that we need six steps to build R from S , six steps to build R from Q , four steps to build Q from S . Thus, we say that S and Q are relatively the most similar. The reason is that S and Q share the common patterns TCTGA and ACGTA.

Therefore, for any two sequences S and Q , we take

$$rd(S, Q) = (c(SQ) - c(S)) + (c(QS) - c(Q)) / c(SQ) + c(QS)$$

as the similarity measure to reflect the similarity degree between S and Q . Then in the above example, $rd(S, Q) = 1/3$ and $rd(S, R) = 13/31$. Our formula also shows that S is more similar to Q than to R . Otu and Sayood [11] have proved that $d(S, Q) = 2 rd(S, Q)$ satisfies the following four conditions:

1. $d(S, Q) \geq 0$
2. $d(S, Q) = d(Q, S)$
3. $d(S, Q) \leq d(S, T) + d(T, Q)$
4. $D(Q, T) + d(S, R) \leq \max \{d(Q, S) + d(T, R), d(Q, R) + d(T, S)\}$

Without question, $rd(S, Q)$ also satisfies the four conditions. Since $d(S, Q)$ does not satisfy that $d(S, Q) = 0$ iff $Q = S$, $rd(S, Q)$ does not satisfy this condition, i.e. $rd(S, Q)$ is not a standard distance metric. However $d(S, Q)$ may be regarded as a distance metric approximately. In other words, if $rd(S, S) \neq 0$, then it can be regarded as an error and ignored, as Otu and Sayood have done when they successfully constructed phylogenetic tree by using mtDNA sequences [11]. The rectified $rd(S, Q)$ can be used to construct hierarchical clustering or phylogenetic tree like other distance metric. We call $rd(S, Q)$ special similarity measure.

(0,1) characteristic sequences of DNA sequences

DNA sequences are composed of four nucleotides. These four nucleotides are arranged linearly on each chain. In research it is usually regarded as a sequence defined over the alphabet $\{A, C, G, T\}$, where the letters represent the four nucleotides bases: adenine, guanine, cytosine, and thymine. Biologists generally classify the four bases into groups according to their chemical structure or the strength of hydrogen bonds. They are as follows: 1. purine $\{A, G\}$ and pyrimidine $\{C, T\}$; 2. amino group $\{A, C\}$ and keto group $\{G, T\}$; 3. weak H-bond $\{A, T\}$ and strong H-bond $\{C, G\}$.

Based on these classifications, any DNA sequence can be transformed into three other sequences. The transformation rules are represented by R, M, W , respectively,

Where

$$R(S(i)) = \begin{cases} 1 & S(i) = A, G \\ 0 & \text{else} \end{cases} \quad M(S(i)) = \begin{cases} 1 & S(i) = A, C \\ 0 & \text{else} \end{cases} \quad W(S(i)) = \begin{cases} 1 & S(i) = A, T \\ 0 & \text{else} \end{cases}$$

For example, for DNA sequence $S=TTACTGAAGCTGAAAGGCGCTGTTCGATCA$, it can be transformed into the following three sequences: $R(S)=001001111001111110100100011001$, $M(S)=001100110100111001010000101011$, $W(S)=111010110010111000001011001101$.

These sequences are called (0,1) characteristic sequences, which are named characteristic sequences of DNA primary sequences in [6]. The (0,1) characteristic sequences are a coarse-grained description of DNA sequences. They provide another chance for analyzing sequences from different aspects.

The weighted measure

Besides DNA primary sequences, (0,1) characteristic sequences may also be used to complement the existing methods for analyzing DNA sequences because they describe DNA sequences from the aspect of the chemistry of four bases. Motivated by this idea, we introduce a weighted similarity measure for the comparison of DNA sequences.

Given n DNA sequences S_1, S_2, \dots, S_n . We firstly transform them into the corresponding (0,1) characteristic sequences, denoted by $R_1, R_2, \dots, R_n; M_1, M_2, \dots, M_n; W_1, W_2, \dots, W_n$. Then according to the rule of production process of sequences and our special similarity measure, we can easily deduce three formulae:

$$\begin{aligned} rd(R_i, R_j) &= (c(R_i R_j) - c(R_i) + c(R_j R_i) - c(R_j)) / (c(R_i R_j) + c(R_j R_i)) \\ rd(M_i, M_j) &= (c(M_i M_j) - c(M_i) + c(M_j M_i) - c(M_j)) / (c(M_i M_j) + c(M_j M_i)) \\ rd(W_i, W_j) &= (c(W_i W_j) - c(W_i) + c(W_j W_i) - c(W_j)) / (c(W_i W_j) + c(W_j W_i)) \end{aligned}$$

Obviously, the above formulae reflect the similarity degree between S_i and S_j from three aspects: 1. from the purine–pyrimidine aspect; 2. from the amino–keto aspect; 3. from the weak–strong H-bond aspect. In other words, $rd(R_i, R_j)$, $rd(M_i, M_j)$, $rd(W_i, W_j)$ reflect the similarity degree between S_i and S_j from the purine–pyrimidine aspect, the amino–keto aspect and weak–strong H-bond aspect, respectively.

Now, we may define the weighted measure as follows:

$$\begin{aligned} wrd(S_i, S_j) &= \xi rd(R_i, R_j) + \eta rd(M_i, M_j) + \zeta rd(W_i, W_j), \\ i, j &= 1, 2, \dots, n. \end{aligned}$$

where ξ, η, ζ are real numbers such that $\xi + \eta + \zeta = 1$.

The virtue of this weighted measure is that it is flexible to operate. The reason is that the three weights are variables and hence can be assigned values according to biologists' different requirements and aims if only they meet the restricted condition. For example, when $\xi = 1, \eta = \zeta = 0$, then $wrd(S_i, S_j) = rd(R_i, R_j)$ and it reflects the similarity degree between S_i and S_j from the purine–pyrimidine aspect; When $\eta = 1, \xi = \zeta = 0$, then $wrd(S_i, S_j) = rd(M_i, M_j)$ and it reflects the similarity degree between S_i and S_j from the amino–keto aspect; When $\zeta = 1, \xi = \eta = 0$, then $wrd(S_i, S_j) = rd(W_i, W_j)$ and it reflects the similarity degree between S_i and S_j from the weak–strong H-bond aspect; When $\xi = \eta = \zeta = 1/3$, then $wrd(S_i, S_j) = (rd(R_i, R_j) + rd(M_i, M_j) + rd(W_i, W_j)) / 3$ and it reflects the full similarity degree between S_i and S_j . As for

an optimal assignment of the three weights, it can be perfectly fulfilled by biologists who know well how much a certain base affects the character of DNA sequence.

In order to compare DNA sequences clearly and systematically, we adopt the form of a matrix to show their similarity relationship, in which the entry is $wrd(S_i, S_j)$, denoted by $WRD = (wrd(S_i, S_j))$, $i=1, 2, \dots, n$ and $j=1, 2, \dots, n$. This matrix is special for the fact that the entries on the main diagonal are not necessarily zeros, which can be deduced directly from the formula $wrd(S, Q)$. However, they are valuable when we use them to scale the similarity degree between two different sequences as illustrated in the next section. So we won't rectify WRD unless we want to construct hierarchical clustering or phylogenetic tree, as described near the end of Section 2.

Results and discussion

In this section, we apply our weighted measure to analyze a set of β -globin genes. At first, we analyze their first exon sequences, whose similarity has been studied by many researchers [4, 7, 8, 12]. The validation of the results is tested by comparing with other results derived by a different method. Table 1 shows the coding sequences of the first exon of these species.

Tables 2, 3, 4 and 5 are obtained by assigning special values to ξ, η and ζ . They show the corresponding similarity results when 1. $\xi = 1, \eta = \zeta = 0$; 2. $\eta = 1, \xi = \zeta = 0$;

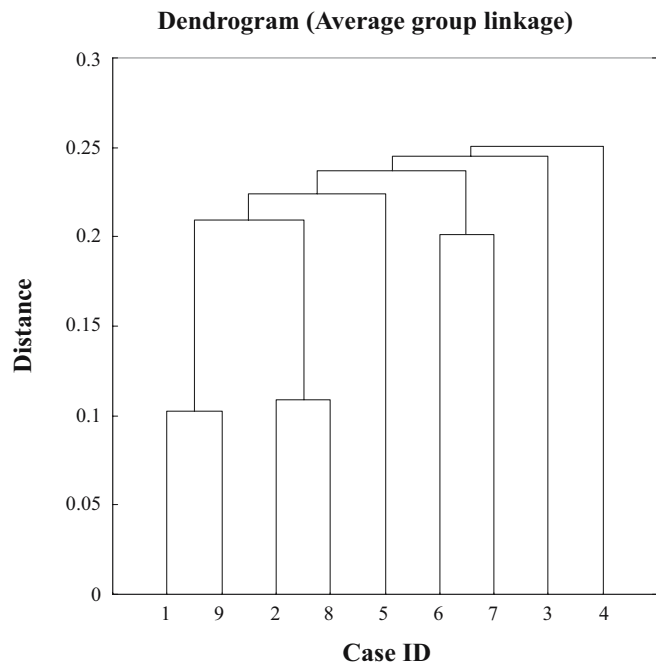


Fig. 1 The dendrogram of the hierarchical clustering of these nine species from the purine–pyrimidine aspect. 1-Human, 2-goat, 3-Opossum, 4-Gallus, 5-Lemur, 6-Mouse, 7-Rat, 8-Bovine, 9-Chimpanzee

Conclusion

The expanding list of genome sequences in various databases has provided a fertile ground for mathematics and computer science. It provides biologists with methods and algorithms to extract information from biological se-

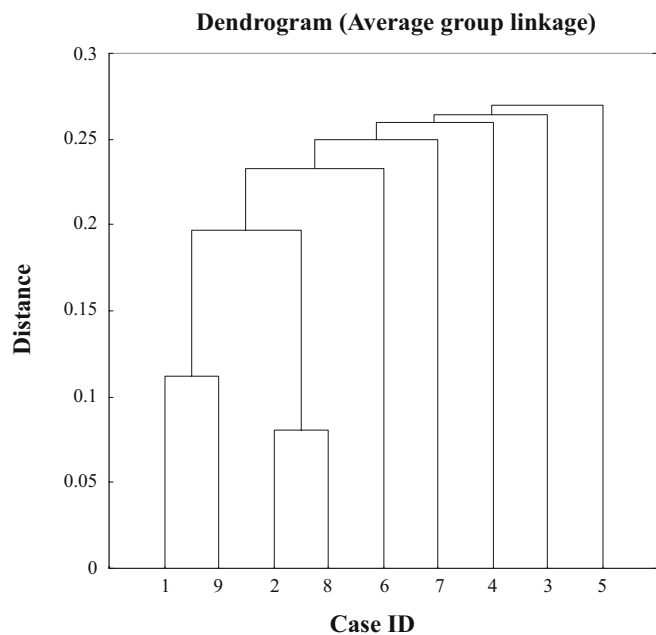


Fig. 2 The dendrogram of the hierarchical clustering of these nine species from the amino–keto aspect. 1-Human, 2-goat, 3-Opossum, 4-Gallus, 5-Lemur, 6-Mouse, 7-Rat, 8-Bovine, 9-Chimpanzee

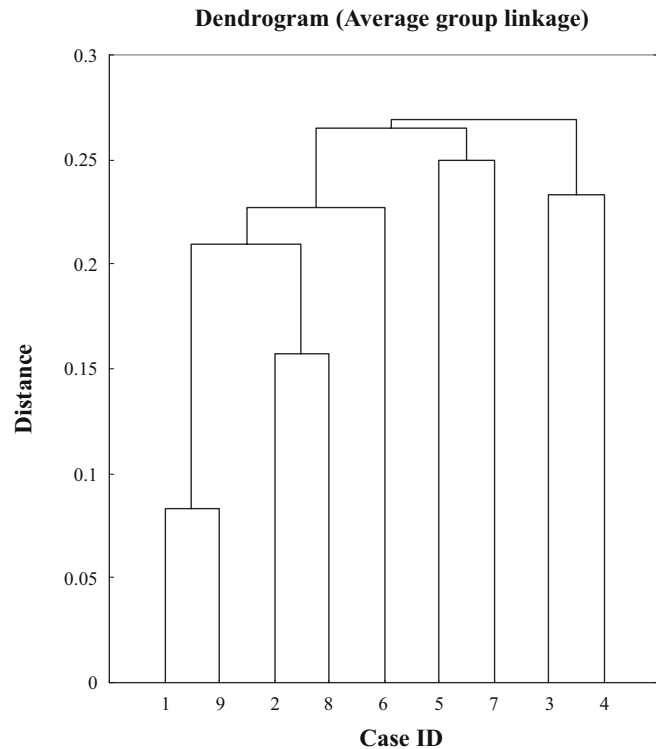


Fig. 3 The dendrogram of the hierarchical clustering of these nine species from the weak H-bond–strong H-bond aspect. 1-Human, 2-goat, 3-Opossum, 4-Gallus, 5-Lemur, 6-Mouse, 7-Rat, 8-Bovine, 9-Chimpanzee

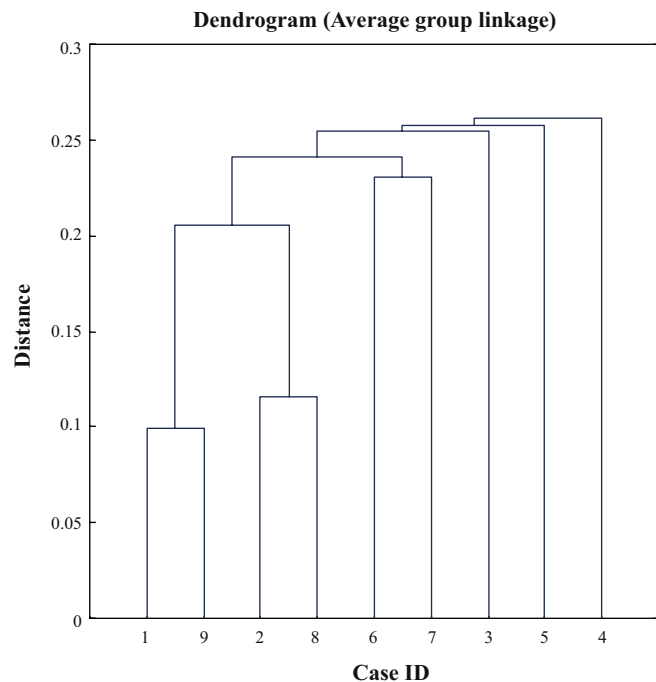


Fig. 4 The dendrogram of the hierarchical clustering of these nine species on the whole. 1-Human, 2-goat, 3-Opossum, 4-Gallus, 5-Lemur, 6-Mouse, 7-Rat, 8-Bovine, 9-Chimpanzee

quences, to analyze biological sequences and to evaluate the results that they obtain. They serve for biological research. In this paper, we present a weighted measure for biologists to analyze the similarity of DNA sequences. This weighted measure brings great flexibility to biologists' research by endowing these weights with variability. By this measure, biologists may assign optimal values to weights to obtain the corresponding optimal analysis result according to their own purpose. For instance, as shown at the end of this paper, this weighted similarity measure may be used to analyze the DNA sequences from three aspects based on different classifications of the four bases according to their chemical structure. It plays a complementary role in the analysis of DNA sequences.

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